o

A simple approach

in solving polynomial equations

-- root 'r' = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$

op-root 'r' = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$

New And Simple Methods

Of

Solving Polynomial Equations

(ALL REAL ROOTS)

(10 new theorems and many new definitions.)

PART -1 (M)

(Appendix)

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This is an appendix

to

‘NEW AND SIMPLE METHODS OF SOLVING POLYNOMIAL EQUATIONS’

(ALL REAL ROOTS)

PART - 1(M)

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SOLVING IRRATIONAL ROOTS.

I have shown here, how to solve polynomial equations, having irrational roots.

Examples.

1) $X^4+7X^3+15X^2+10X+2=0$
2) $X^3+20X^2+123X+230=0$

Example 1.

1) $X^4+7X^3+15X^2+10X+2=0$ --------------- equation 1.

Using theorem 1.

The smallest op-root

$$rs > \frac{K}{Z}$$

Z =10 K = 2

$$rs > \frac{2}{10} = 0.2$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{4\times2}{10} = 0.8$$

The smallest op-root rs is between 0.2 and 0.8 (including this number).

In graph;

Op- roots scale;

0	¼	½	¾	1
0.2	0.35	0.5	0.65	0.8

$$\frac{A}{n} = \frac{7}{4} = 1.75$$

$$\frac{nK}{z} = 0.8$$

$$\frac{A}{n} \neq \frac{nK}{z}$$

So, the smallest op-root is less than the full value.

Diminish the op-roots by ¾ of the value (= 0.65).

0.65	1	7	15	10	2
×	0	0.65	4.1275	7.0671	1.9064
	1	6.35	10.8725	2.9329	0.0936
	0	0.65	3.705	4.6589	
	1	5.7	7.1675	-- 1.7260	

Don't proceed further. Since minus sign appeared. Mark × sign as it is cancelled.

Diminish the roots by ½ of the value(=0.5)

0.5	1	7	15	10	2
×	0	0.5	3.25	5.875	2.0625
	1	6.5	11.75	4.125	-- 0.0625

Don't proceed further. Since minus sign appeared.

Diminish the op-roots by 1/4 of the value(= 0.35)

0.35	1	7	15	10	2
	0	0.35	2.3275	4.4354	1.9476
	1	6.65	12.6725	5.5646	0.0524
	0	0.35	2.2050	3.6636	
	1	6.30	10.4675	1.9010	
	0	0.35	2.0825		
	1	5.95	8.3850		
	0	0.35			
	1	5.60			

Part of the op-root $r_1 = r_{11} = 0.35$

The new equation is

$X^4+5.6X^3+8.3850X^2+1.9010X+0.0524=0$ --------------- equation 2.

Using theorem 1,

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z = 1.9010 K = 0.0524

$$rs > \frac{0.0524}{1.9010} = 0.0275$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{4\times0.0524}{1.9010} = 0.1104$$

The smallest op-root rs is between 0.0275and 0.1104 (including this number).

In graph;

Op- roots scale;

0 ¼ ✕ ½ ✕ ¾ ✕ 1

|----------------------|----------------------|----------------------|----------------------|

0.0275 0.0483 0.1104

As per theorem 5, The smallest op-root rs must be less than ½ of the value .

Diminish the op-roots by 1/4 of the value. (=0.0483)

0.0483	1	5.6	8.3850	1.9010	0.0524
×	0	0.0483	0.2681	0.3920	0.0729
	1	5.5517	8.1169	1.5090	--0.0205

Don't proceed further. Since minus sign appeared.

Diminish the op-roots by the starting value (= 0.0275)

0.0275	1	5.6	8.3850	1.9010	0.0524
	0	0.0275	0.1532	0.2264	0.0461
	1	5.5725	8.2318	1.6746	0.0063
	0	0.0275	0.1525	0.2222	
	1	5.5450	8.0793	1.4524	
	0	0.0275	0.1517		
	1	5.5175	7.9276		
	0	0.0275			
	1	5.4900			

Another part of the op-root $r_1 = r_{12} = 0.0275$

The new equation is

$X^4+5.49X^3+7.9276X^2+1.4524X+0.0063=0$ --------------- equation 3.

Using theorem 1.

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z = 1.4524 K = 0.0063

$$rs > \frac{0.0063}{1.4524} = 0.0043$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{4 \times 0.0063}{1.4524} = 0.0176$$

The smallest op-root rs is between 0.0043and 0.0176 (including this number).

In graph;

Op- roots scale;

0 ×¼ ×½ ×¾ × 1

|---------------------|-----------------------|------------------|---------------------|

0.0043 0.0176

As per theorem 5, the smallest op-root rs must be less than ¼ of the value.

Diminish the op-roots by the starting value (= 0.0043).

0.0043	1	5.4900	7.9276	1.4524	0.0063
	0	0.0043	0.0236	0.0340	0.0061
	1	5.4857	7.9040	1.4184	0.0002
	0	0.0043	0.0236	0.0339	
	1	5.4814	7.8804	1.3845	
	0	0.0043	0.0236		
	1	5.4771	7.8568		
	0	0.0043			
	1	5.4728			

`

Another part of op-root $r_1 = r_{13} = 0.0043$

The new equation is

$X^4+5.4728X^3+7.8568X^2+1.3845X+0.0002=0$ --------------- equation 4.

Using theorem 1.

The smallest op-root

$$rs > \frac{K}{Z}$$

Z = 1.3845 K = 0.0002

rs $> \frac{0.0002}{1.3845}$ = 0.0001

Another part of the op-root r_1 = r_{14} = 0.0001

The op-root r_1

= $r_{11}+r_{12}+r_{13}+r_{14}$

= 0.35+0.0275+0.0043+0.0001

r_1 = <u>0.3819</u>

The answer is in numerical (in decimal), to get the exact answer, we have to change the answer to the basic form.

<u>The basic form of the smallest root of an equation (see theorem 3.)</u>

$$= \frac{A-\sqrt{E}}{n}$$

Where **A** is the co-efficient of X^{n-1}, **n** is the highest power of X, and **E** is the number which decides the value of the smallest root.

So,

$$0.3819 = \frac{A-\sqrt{E}}{n}$$

A = 7 n = 4

$$0.3819 = \frac{7-\sqrt{E}}{4}$$

$$\sqrt{E} = 5.4724$$

$$E = 29.9472$$

$$0.3819 = \frac{7- \sqrt{29.9472}}{4}$$

Since A is a small number, a small adjustment is enough to get the exact answer.

$$0.3819 = \frac{(7—1\ ) - \sqrt{29.9472} + 1}{4}$$

$$= \frac{6 - (\sqrt{29.9472} - 1)}{4}$$

$$= \frac{6 - \sqrt{(\sqrt{29.9472} - 1)\text{^}2}}{4}$$

$$= \frac{6- \sqrt{20.0024}}{4}$$

$\sqrt{20.0024}$ might be $\sqrt{20}$.

So,

$$0.3819 = \frac{6- \sqrt{20.}}{4} = = \frac{3- \sqrt{5}}{2}$$

The exact answer of the op-root r_1 $= \mathbf{\frac{3- \sqrt{5}}{2}}$

The exact answer of the op-root r_2 (pair of op-root r_1)

$$= \mathbf{\frac{3+ \sqrt{5}}{2}}$$

<u>Verifying;</u>

$\frac{3-\sqrt{5}}{2}$	1	7	15	10	2
	0	$\frac{3-\sqrt{5}}{2}$	$7-2\sqrt{5}$	$7-\sqrt{5}$	2
$\frac{3+\sqrt{5}}{2}$	1	$\frac{11+\sqrt{5}}{2}$	$8+2\sqrt{5}$	$3+\sqrt{5}$	0
	0	$\frac{3+\sqrt{5}}{2}$	$6+2\sqrt{5}$	$3+\sqrt{5}$	
	1	4	2	0	

The new equation is

$X^2+4X+2=0$ --------------- equation 5.

<u>Using quadratic equation formula,</u>

op-root r_3 = $\mathbf{2-\sqrt{2}}$ op-root r_4 = $\mathbf{2+\sqrt{2}}$

The op-roots are

= $\frac{3-\sqrt{5}}{2}$, $\frac{3+\sqrt{5}}{2}$, $2-\sqrt{2}$, and $2+\sqrt{2}$.

The roots are

= -- $\left(\frac{3-\sqrt{5}}{2}\right)$, --$\left(\frac{3+\sqrt{5}}{2}\right)$, $-(2-\sqrt{2})$, and -- $(2+\sqrt{2})$.

Example 2.

$X^3+20X^2+123X+230=0$-------------- equation 1.

Using theorem 1.

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z =123   K = 230

$$rs > \frac{230}{123} = 1.8699$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{3 \times 230}{123} = 5.61$$

The smallest op-root rs is between 1.8699 and 5.61(including this number).

In graph;

Op- root scale;

0	¼	½	¾	1
1.8699	2.8050	3.74	4.675	5.61

$$\frac{A}{n} = \frac{20}{3} = 6.6666$$

$$\frac{nK}{z} = 5.61$$

$$\frac{A}{n} \neq \frac{nK}{z}$$

So, the smallest op-root is less than the full value.

Diminish the op-roots by ¾ of the value(= 4.675).Round it off to 5.

5	1	20	123	230
×	0	5	75	240
	1	15	48	-- 10

Don’t proceed further. Since minus sign appeared. Mark as × sign as it is cancelled.

Diminish the roots by ½ of the value(=3.74).Round it off to 4.

4	1	20	123	230
×	0	4	64	236
	1	16	59	-- 6

Don’t proceed further. Since minus sign appeared.

Diminish the op-roots by ¼ of the value(= 2.8050) round it off to 3.

3	1	20	123	230
	0	3	51	216
	1	17	72	14
	0	3	42	
	1	14	30	
	0	3		
	1	11		

Part of the op-root $r_1 = r_{11} = 3$

The new equation is

$X^3+11X^2+30X+14=0$ --------------- equation 2.

Using theorem 1.

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z = 30 K = 14

$$rs > \frac{14}{30} = 0.4666$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{3\times14}{30} = 1.4001$$

The smallest op-root rs is between 0.4666 and 1.4001 (including this number).

<u>In graph</u>;

<u>Op- root scale;</u>

```
0                    ¼             × ½               × ¾             × 1

|--------------------|---------------------|---------------------|-------------------|

0.4666            0.7                                                      1.4001
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<u>As per theorem 5</u>, the smallest op-root rs must be less than ½ of value.

Diminish the roots by ¼ of the value(=0.7).

0.7	1	11	30	14
×	0	0.7	7.21	15.953
	1	10.3	22.79	-- 1.953

Don't proceed further. Since minus sign appeared.

Diminish the op-roots by the starting value(=0.4666)

0.4666	1	11	30	14
	0	0.4666	4.9149	11.7047
	1	10.5334	25.0851	2.2953
	0	0.4666	4.6972	

1	10.0668	20.3879
0	0.4666	
1	9.6002	

Another part of the op-root $r_1 = r_{12} = 0.4666$

The new equation is

$X^3+9.6002X^2+20.3879X+2.2953=0$ --------------- equation 3.

<u>Using theorem 1,</u>

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z = 20.3879 K = 2.2953

$$rs > \frac{2.2953}{20.3879} = 0.1125$$

$$rs \leq \frac{nK}{Z}$$

$$rs \leq \frac{3\times2.2953}{20.3879} = 0.3378$$

The smallest op-root rs is between 0.1125and 0.3378 (including this number).

In graph;

Op- root scale;

0 × ¼ × ½ × ¾ × 1

|-------------------|-----------------------|---------------------|-----------------------|

0.1125 0.3378

Diminish the op-roots by the starting value. (=0.1125)

0.1125	1	9.6002	20.3879	2.2953
	0	0.1125	1.0674	2.1736
	1	9.4877	19.3205	0.1217
	0	0.1125	1.0547	
	1	9.3752	18.2658	
	0	0.1125		
	1	9.2627		

Another part of op-root $r_1 = r_{13} = 0.1125$

The new equation is

$X^3+9.2627X^2+18.2658X+0.1217=0$ --------------- equation 4.

Using theorem 1,

The smallest op-root,

$$rs > \frac{K}{Z}$$

Z = 18.2658 K = 0.1217

$$rs > \frac{0.1217}{18.2658} = 0.0067$$

Another part of op-root r_{14} = 0.0067

The smallest op-root

$r_1 = r_{11}+r_{12}+r_{13}+r_{14}$

= 3+0.4666+0.1125+0.0067

= 3.5858

As A is a big number, if we adjust the numerical answer to get the exact answer,it may take longer time. So ,first, we have to find all the roots in numerical value.

3.5858	1	20	123	230
	0	3.5858	58.8580	230.00
	1	16.4142	64.142	0

The new equation is

$X^3+16.4142X^2+64.142=0$ --------------- equation 5

Using quadratic equation formula,

The op-root r_2 = 6.4142

The op-root r_3 = 10

The pair op-roots are r_1 and r_2

(since $r_1 + r_2 = 3.5858 + 6.4142 = 10$)

GETTING THE EXACT ANSWER.

The smallest op-root = 3.5858

The answer is in numerical (in decimal),to get the exact answer we have to change the answer to basic form.

The basic form of the smallest root of an equation (see theorem 3).

$$= \frac{A - \sqrt{E}}{n}$$

Where A is the co-efficient of X^{n-1} ,n is the highest power of X., and E is the number which decide the value of a smallest root.

The pair roots are r_1 and r_2

(since $r_1 + r_2 = 3.5858 + 6.4156 = 10$)

$$3.5858 = \frac{A - \sqrt{E}}{n}$$

Here A = 10 and n = 2

$$3.5858 = \frac{10 - \sqrt{E}}{2}$$

$\sqrt{E}$ = 2.8284

E = 7.9998

E might be 8.

SO,

$r_1 = 3.5858 = \frac{10-\sqrt{8}}{2}$

$\mathbf{r_1 = 5 - \sqrt{2}}$

and $\mathbf{r_2 = 5 + \sqrt{2}}$

Verifying

$5 - \sqrt{2}$	1	20	123	230
	0	$5 - \sqrt{2}$	$73 - 10\sqrt{2}$	230
$5 + \sqrt{2}$	1	$15 + \sqrt{2}$	$50 + 10\sqrt{2}$	0
	0	$5 + \sqrt{2}$	$50 + 10\sqrt{2}$	
	1	10	0	

The op-root $\mathbf{r_3}$ **= 10.**

The op-roots are

= $\mathbf{5 - \sqrt{2}, \ 5 + \sqrt{2}}$ **and 10 .**

The roots are

= **-- (** $\mathbf{5 - \sqrt{2}}$ **) , -- (** $\mathbf{5 + \sqrt{2}}$ **) and --10.**

By using this method, all the irrational roots of an equation can be solved very easily.

---End.---.