Finding the approximate roots values of a real roots polynomial equation.

Theorem 1. Sum of roots.

In an equation,

$A_n X^n + A_{n-1}X^{n-1}$.......... $A_3X^3+A_2X^2+A_1X^1+A_0 = 0$,

Whose roots are real and negative. A_1, A_2, A_3 .... A_{n-1}, A_n are the coefficients of

X^1, X^2, X^3.......... X^{n-1}, X^n respectively. n is the highest power of the equation.

$A_n = 1$, A_0 is a constant.

-a, -b, -c, -d ....are the roots of the equation in ascending order.

In this equation,

1. $a > \frac{A0}{A1}$ & $a \le n \times \frac{A0}{A1}$
2. $a+b > \frac{A1}{A2}$ & $a+b \le (n-1)\frac{A1}{A2}$
3. $a+b+c > \frac{A2}{A3}$ & $a+b+c \le (n-2)\frac{A2}{A3}$

And so on.

....

Theorem 2. Multiplication of roots.

In an equation

$A_n X^n + A_{n-1}X^{n-1}$.......... $A_3X^3+A_2X^2+A_1X^1+A_0 = 0$,

Whose roots are real and negative. A_1, A_2, A_3 .... A_{n-1}, A_n are the coefficients of

X^1, X^2, X^3.......... X^{n-1}, X^n respectively. n is the highest power of the equation.

$A_n = 1$, A_0 is a constant.

-a, -b, -c, -d ....are the roots of the equation in ascending order.

In this equation,

1. a > $\frac{A0}{A1}$ & a ≤ $nc1\frac{A0}{A1}$
2. ab > $\frac{A0}{A2}$ & ab ≤ $nc2\frac{A0}{A2}$
3. abc > $\frac{A0}{A3}$ & abc ≤ $nc3\frac{A0}{A3}$

And so on.

The values of nc1,nc2, nc3 .... are the corresponding values in Pascal's pyramid.

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Example 1.

$X^3+10X^2+31X+30 = 0$

(The roots are -2,-3,-5.)

$A_0=30$, $A_1 = 31$, $A_2 = 10$, $A_3 = 1$, n = 3.

As per theoram 1 ,

1. a > $\frac{A0}{A1}$ & a ≤ n $\frac{A0}{A1}$

   a > $\frac{30}{31} = 0.96$ & a ≤3× 0.96= 2.88

   a is between 0.96 and 2.88 which is correct. (a = 2)

2. a+b > $\frac{A1}{A2}$ & a+b ≤ $(n-1)\frac{A1}{A2}$

   a+b > $\frac{31}{10}$=3.1 & a+b ≤ 2×3.1 = 6.2

   a+b is between 3.1 and 6.2 which is correct (a+b =2+3 = 5)

3. a+b+c > $\frac{A2}{A3}$ & a+b+c ≤ $(n-2)\frac{A2}{A3}$

   a+b+c > $\frac{10}{1}$= 10 & a+b+c ≤ 1 ×10 = 10

   a+b+c is 10, which is correct (a+b+c =2+3+5 = 10)

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As per theoram 2,

1. a > $\frac{A0}{A1}$ & a ≤ $nc1\frac{A0}{A1}$

   a > $\frac{30}{31}$ = 0.96 & a ≤ 3× 0.96 = 2.88

   a is between 0.96 and 2.88 which is correct. (a = 2)

2. ab > $\frac{A0}{A2}$ & ab ≤ $nc2\frac{A0}{A2}$

   ab > $\frac{30}{10}$ = 3 & ab ≤ 3×3 =9

   ab is between 3 and 9which is correct. (ab = 2×3 = 6)

3. abc > $\frac{A0}{A3}$ & abc ≤ $nc3\frac{A0}{A3}$

   abc > $\frac{30}{1}$ = 30 & abc ≤ 1×30 = 30

   abc is 30, which is correct. (abc = 2×3×5 = 30)

<u>Pascal's pyramid</u>

1   nc1 = 3, nc2 = 3, nc3 = 1.

1 2 1

<u>1 3 3 1</u>

1 4 6 4 1

1 5 10 10 5 1

....

Example 2.

$X^4+30X^3+301X^2+1140X+1300 = 0$,

(The roots are -2, -5,-10,-13.)

A_0=1300, A_1 = 1140, A_2 = 301, , A_3 = 30, A_4 = 1, n = 4.

As per theoram 1 ,

1. a > $\frac{A0}{A1}$ & a ≤ n $\frac{A0}{A1}$

   a > $\frac{1300}{1140} = 1.14$ & a ≤ 4× 1.14 = 4.56

   a is between 1.14 and 4.56 which is correct. (a = 2)

2. a+b > $\frac{A1}{A2}$ & a+b ≤ $(n-1)\frac{A1}{A2}$

   a+b > $\frac{1140}{301}$= 3.78 & a+b ≤ 3×3.78 = 11.34

   a+b is between 3.78 and 11.34 which is correct (a+b = 2+5=7)

3. a+b+c > $\frac{A2}{A3}$ & a+b+c ≤ $(n-2)\frac{A2}{A3}$

   a+b+c > $\frac{301}{30}$=10.03 & a+b+c ≤ 2×10.03 = 20.06

   a+b+c is between 10.03 and 20.06 which is correct (a+b+c = 2+5+10 = 17)

4. a+b+c+d > $\frac{A3}{A4}$ & a+b+c+d ≤ $(n-3)\frac{A3}{A4}$

   a+b+c+d > $\frac{30}{1}$= 30 & a+b+c+d ≤ 1× 30 = 30

   a+b+c+d is 30 which is correct (a+b+c+d = 2+5+10+13 = 30)

As per theoram 2,

1. a > $\frac{A0}{A1}$ & a ≤ nc1 $\frac{A0}{A1}$

   a > $\frac{1300}{1140} = 1.14$ & a ≤4× 1.14=4.56

   a is between 1.14 and 4.56 which is correct. (a = 2)

2. ab > $\frac{A0}{A2}$ & ab ≤ $nc2\frac{A0}{A2}$

   ab > $\frac{1300}{301}$ = 4.31 & ab ≤ 6× 4.31 = 25.86

   ab is between 4.31 and 25.86 which is correct. (ab = 2× 5 = 10)

3. abc > $\frac{A0}{A3}$ & abc ≤ $nc3\frac{A0}{A3}$

   abc > $\frac{1300}{30}$= 43.33 & abc ≤ 4× 43.33 = 173.32

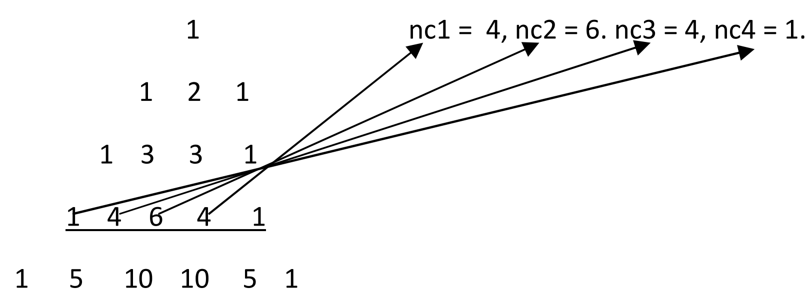
   abc is between 43.33 and 173.32 which is correct (abc = 2×5×10 =100)

4. abcd > $\frac{A0}{A4}$ & abcd ≤ $nc4\frac{A0}{A4}$

   abcd > $\frac{1300}{1}$= 1300 & abcd ≤ 1× 1300 = 1300

   abcd is 1300 which is correct (abcd = 2×5×10×13 =1300)

Pascal pyramid



Solving all roots of an equation at one calcuation.

Example.

$X^3 + 10x^2 + 31x + 30 = 0$ ....equation 1.

Sqrare the roots of the equation 4 times to form new equation.

$X^3 + 38x^2 + 36x + 900 = 0$ ....equation 2.

$X^3 + 722x^2 + 61921x + 900 = 0$ ....equation 3.

$X^3 + 397442x^2 + 26645702\times10^2 x + 65610000\times10^4 = 0$ ....equation 4.

$X^3 + 15233100\times10^4 x^2 + 657841117\times10^{10}x + 43046721\times 10^{16} = 0$ ....equation 5.

Solving equation 5.

$X^3 + 15233100\times10^4 x^2 + 657841117\times10^{10}x + 43046721\times 10^{16} = 0$ ....equation 5.

As per Theorem 1 (sums of roots),

$a > \frac{A0}{A1}$ & $a \le n\times\frac{A0}{A1}$

$A_0 = 43046721\times 10^{16}$, $A_1 = 657841117\times10^{10}$, $A_2 = 15233100\times10^4$, $A_3 = 1$, $n = 3$.

a of equation 5 $> \frac{43046721\times 10\text{^}16,}{657841117\times10\text{^}10} = 65436$ (a of equation 5 $\le 3 \times 65436 = 196308$)

a of equation 1 > $(65436)^{1/16}$ =1.99 ≅ 2. (a of equation 1 ≤ $(196308)^{1/16}$= 2.14)

-2 is the root of equation 1. (and verified)

2. a+b > $\frac{A1}{A2}$ & a+b ≤ $(n-1)\frac{A1}{A2}$

a+b of equation 5 > a+b > $\frac{657841117\times 10^{\wedge}10}{15233100\times 10^{\wedge}4}$ = 43184979 (a+b of equation 5 ≤ 2× 43184979 = 86369959)

b of equation 5 > 43184979 – 65436 = 43119543 (b of equation 5≤ 86369959 – 196308 = 86173651)

b of equation 1 > b > $(43119543)^{1/16}$ = 3.00 (b of equation 1 ≤ $(86173651)^{1/16}$ = 3.13)

-3 is a root of equation 1 and verified.

C = 10 – 2 -3 = 5

-5 is root of the equation 1.

The roots of equation 1 are -2,-3,-5.

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