o

A breakthrough

-- root 'r' = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$ **in solving** op-root 'r' = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$

polynomial equations

New And Simple Methods

Of

Solving Polynomial Equations

(ALL REAL ROOTS)

(10 new theorems and many new definitions.)

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CONTENTS

1.INTRODUCTION

I have created more than 10 new theorems and many new definitions that all are related to polynomial equations.

Using these theorems, Polynomial equation whose **all roots are real**, can be solved very easily and quickly. It is very simple.

My theories says, if a polynomial equation's **all roots are real,**

The smallest root of the equation is between

$$-- \left(\frac{A+(n-1)\sqrt{A^2-TB}}{n} \right) \quad \text{and} \quad -- \left(\frac{A+\sqrt{A^2-TB}}{n} \right)$$

The biggest root of the equation is between

$$-- \left(\frac{A-\sqrt{A^2-TB}}{n} \right) \quad \text{and} \quad -- \left(\frac{A-(n-1)\sqrt{A^2-TB}}{n} \right)$$

(Where A and B are the co-efficient of X^{n-1} and X^{n-2} respectively, T = $\frac{2n}{n-1}$, And n is the highest power of X.)

Example.

$X^3 + 2X^2 -- 13X + 10 = 0$

The smallest root is between

-- ($\frac{2+(3-1)\sqrt{2^2-(\,3\times -13\,)}}{3}$) and -- ($\frac{2+\sqrt{2^2-(\,3\times -13\,)}}{3}$)

The smallest root is between **-- 5.0382 and -- 2.8524**

The biggest root is between

-- ($\frac{2-\sqrt{2^2-(\,3\times -13\,)}}{3}$) and -- ($\frac{2-(3-1)\sqrt{2^2-(\,3\times -13\,)}}{3}$)

The biggest root is between **1.5191 and 3.7049**

(Detailed explanations, shown in the coming chapters.)

It is so simple.

I explained here only few methods of solving polynomial equations. There are a lot of other options and other methods available using these theorems.

These theorems are checked, many times thoroughly and proved correct. If any fault is found in these theorems, it is probably due to wrong words or grammar or spelling or typing mistakes, or bad presentation.

I hope these theorems may also shed some light on **'Abel's Impossibility Theorem'**.

Though I didn't try to solve **imaginary roots equation**, I hope these theorems may help to solve imaginary roots equation also.

INPORTANT:

OP-ROOT:

To simplify my writings, and to avoid confusion, I coin a new word **'op-root'** .

Let us call **'root with opposite sign'** as **'op-root'.**

ROOT		**OP-ROOT**
5	→	-- 5
-- 2	→	2
a	→	-- a
--b	→	b
The smallest root	→	The biggest op-root
The biggest root	→	The smallest op-root

----

2. THEOREM 1 to 5

(The Smallest Op-root)

Theorem –1

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X. (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than $\frac{K}{Z}$ and less than or equal to $n\frac{K}{Z}$. *(The smallest op-root (rs) = more precisely, the op-root which is not bigger than any other op-root in the equation.)*

$$rs > \frac{K}{Z}$$

& $$rs \leq n\frac{K}{Z}$$

Example

$$X^3 + 7X^2 + 14X + 8 = 0$$

$$rs > \frac{K}{Z}$$

Z = 14 K = 8

$$rs > \frac{8}{14}$$

> 0.5714

$$rs \leq n\frac{K}{Z}$$

Z = 14 k = 8 n = 3

$$rs \leq 3 \times \frac{8}{14}$$

$\leq$ **1.7142**

The smallest op-root (rs) is between **0.5714** and **1.7142**(including this number).

Theorem – 2.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X. (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than $\frac{K}{Z-Y\frac{K}{Z}}$ and less than or equal to $\frac{K}{Z-2Y\frac{K}{Z}}$.

$$rs > \frac{K}{Z-Y\frac{K}{Z}}$$

$$\& \quad rs \leq \frac{K}{Z-2Y\frac{K}{Z}}$$

Example

$X^3 + 10X^2+31X+30 = 0$

$$rs > \frac{K}{Z-Y\frac{K}{Z}}$$

Y = 10 Z = 31 K = 30

$$rs > \frac{30}{31-10\times\frac{30}{31}}$$

> **1.4069**

$$rs \le \frac{K}{Z-2Y\frac{K}{Z}}$$

Y = 10 Z = 31 K = 30

$$rs \le \frac{30}{31-2\times10\times\frac{30}{31}}$$

$$\le \underline{\mathbf{2.5761}}$$

The smallest op-root (rs) is between **<u>1.4069</u>** and **<u>2.5761</u>**(including this number)

<u>**Theorem – 3**</u>

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real (positive or negative or both)**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than or equal to $\frac{A-(n-1)\sqrt{A^2-TB}}{n}$ and less than or equal to $\frac{A-\sqrt{A^2-TB}}{n}$.

{ THE VALUE ‘ T ’ = $\frac{2n}{n-1}$ }

$$rs \ge \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

& $$rs \le \frac{A-\sqrt{A^2-TB}}{n}$$

Example X^3 -- X^2 -- 8X+12 = 0

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

A = -- 1 B = -- 8 n = 3 $T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

$$rs \geq \frac{-1-(3-1)\sqrt{(-1^2)-(3\times-8)}}{3}$$

$\geq$ **-- 3.6666**

$$rs \leq \frac{A-\sqrt{A^2-TB}}{n}$$

A = -- 1 B = -- 8 n = 3 T = 3

$$rs \leq \frac{-1-\sqrt{(-1^2)-(3\times-8)}}{3}$$

$\leq$ **-- 2**

The smallest op-root (rs) is between **-- 3.6666**and **-- 2** (including both numbers)

Theorem –4

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2},

X^2 and X respectively. K is a constant. And n is the highest power of X.(n = Positive whole number & > 1.).

The smallest op-root (rs) of the above equation is more than or equal to $\frac{nK}{Z+(n-1)\sqrt{Z^2-TYK}}$ and less than or equal to $\frac{nK}{Z+\sqrt{Z^2-TYK}}$.

{ THE VALUE 'T' = $\frac{2n}{n-1}$ }

$$rs \geq \frac{nK}{Z+(n-1)\sqrt{Z^2-TYK}}$$

$$\& \quad rs \leq \frac{nK}{Z+\sqrt{Z^2-TYK}}$$

<u>Example</u>

$$X^3 + 10X^2+31X+30 = 0$$

$$rs \geq \frac{nK}{Z+(n-1)\sqrt{Z^2-TYK}}$$

Y = 10 Z = 31 K = 30 n = 3 T = $\frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

$$rs \geq \frac{3\times30}{31+(3-1)\sqrt{31^2-3\times10\times30}}$$

$$\geq \underline{\mathbf{1.9304}}$$

$$rs \leq \frac{nK}{Z+\sqrt{Z^2-TYK}}$$

Y = 10 Z = 31 K = 30 n = 3 T = 3

$$rs \leq \frac{3\times30}{31+\sqrt{31^2-3\times10\times30}}$$

$$\leq \underline{\mathbf{2.3189}}$$

The smallest op-root (rs) is between **1.9304** and **2.3189** (including both numbers)

Theorem—5

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X .(n = Positive whole number & > 1.)

Effective ratio(NrsR) of the smallest op-root (Nrs) of a new equation, which is obtained by diminishing the op-roots of the above original equation, by a positive number,(which is less than the smallest op-root(rs) of the original equation), is less than **Effective Ration (rsR)** of the smallest op-root (rs) of the original equation**(Effective Ratio – see chapter 8.' Definitions').**

NrsR $<$ rsR

Example

$X^3 + 10X^2 +31X + 30 = 0$

The op-roots are 2,3, and 5

Effective Ratio of op-root ‘a’ = $(\frac{1}{b} + \frac{1}{c}) \times \frac{a}{n-1}$

Effective Ratio (rsR) of the smallest op-root (rs) ‘2’ = $(\frac{1}{3} + \frac{1}{5}) \times \frac{2}{3-1}$

rsR = **0.5333**

Diminish the op-roots of the equation by 1.

1	1	10	31	30
	0	1	9	22
	1	9	22	8
	0	1	8	
	1	8	14	
	0	1		
	1	7		

The new equation is $X^3 + 7X^2 + 14X + 8 = 0$

The op-roots of the new equation are 1, 2 and 4.

Effective ratio (NrsR) of the smallest op-root (Nrs)

of the new equation = $(\frac{1}{2} + \frac{1}{4}) \times \frac{1}{3-1}$

. NrsR = **0.3750**

0.3750 < 0.5333

NrsR < rsR

----

3. SOLVING POLYNOMIAL EQUATION (1.)

(Some methods.)

Using theorem 1,3 and 5, we can find the roots of polynomial equation whose **all roots are real**. We can use other theorems too.

The method **'Bottom value approach method',** which I used here, is to solve **very simple equations.**

'Bottom value approach method' is not the best method of solving equations. But it is enough to solve 'very simple equations'. The best method, **'From top to bottom value approach method'** is explained in next chapter, 'Solving Polynomial Equation 2'.

The following simple equations are solved here, using theorems 1,3 and 5 and **' Bottom value approach method'**. The answer is in numerical value.

1. $X^3+5X^2+8X+4=0$
2. $X^4+11X^3+41X^2+61X+30=0$
3. $X^2+98X+97=0$
4. $X^3—2X^2—X+2=0$
5. $X^5+10X^4+40X^3+80X^2+80X+32=0$

Example-1.

$X^3+5X^2+8X+4=0$ -------------equation 1.

(Using theorem 1.)

$$rs > \frac{K}{Z}$$

Z = 8 K = 4

$$rs > \frac{4}{8}$$

$$> \mathbf{0.5}$$

$$rs \leq n\frac{K}{Z}$$

Z = 8 K=4 n = 3

$$rs \leq 3 \times \frac{4}{8}$$

$$\leq \mathbf{1.5}$$

The smallest op-root (rs) is between **0.5** and **1.5**(including this number). The smallest whole number between them is 1.

Diminish the op-roots by 1.

1	1	5	8	4
	0	1	4	4
	1	4	4	0
	0	1	3	
	1	3	1	

0	1
1	2

The op-root r_1 is **1.**

The new equation is

$X^2+2X+1=0$ ------------------- equation 2.

Part of op-root $r_2 = r_{21}$ Part of op-root $r_3 = r_{31}$

Using quadratic equation formula,

$r_{21} = \mathbf{1}$ $r_{31} = \mathbf{1}$

$r_2 = r_1 + r_{21}$

$= 1 + 1$

$= \mathbf{2}$

$r_3 = r_1 + r_{31}$

$= 1 + 1$

$= \mathbf{2}$

The op-roots are r_1 = **1,** r_2 = **2** r_3 = **2**

The roots are = **--1, --2** , **--2 .**

Example 2.

$X^4+11X^3+41X^2+61X+30=0$ ------------equation1.

(Using theorem 1.)

$$rs > \frac{K}{Z}$$

Z = 61 K = 30

$$rs > \frac{30}{61}$$

$$> \mathbf{0.4918}$$

$$rs \le n\frac{K}{Z}$$

Z = 61 K = 30 n = 4

$$rs \le 4 \times \frac{30}{61}$$

$$\le \mathbf{1.9672}$$

The smallest op-root (rs) is between **0.4918** and **1.9672**(including this number). The smallest whole number is 1.

Diminish the op-roots by 1.

1	1	11	41	61	30
	0	1	10	31	30
	1	10	31	30	0
	0	1	9	22	
	1	9	22	8	
	0	1	8		
	1	8	14		

0	1
1	7

The op-root r_1 is **1**.

The new equation is

$X^3+7X^2+14X+8=0$ ------------equation 2.

$$rs > \frac{K}{Z}$$

Z = 14 K = 8

$$rs > \frac{8}{14}$$

$$> \mathbf{0.5714}$$

$$rs \leq n\frac{K}{Z}$$

Z = 14 K = 8 n = 3

$$rs \leq 3 \times \frac{8}{14}$$

$$\leq \mathbf{1.7142}$$

The smallest op-root (rs) is between **0.5714** and **1.7142**.(including this number).The smallest whole number is **1**.

Diminish the op-roots by 1.

1	1	7	14	8
	0	1	6	8

1	6	8	0
0	1	5	
1	5	3	
0	1		
1	4		

Part of op-root $r_2 = r_{21} = 1$

The op-root $r_2 = r_1 + r_{21} = 1 + 1$

$= \underline{\mathbf{2}}$

The new equation is

$X^2+4X+3=0$ -------------equation 3.

Part of op-root $r_3 = r_{31}$ Part of op-root $r_4 = r_{41}$

Using quadratic equation formula,

$r_{31} = 1$ $r_{41} = 3$

The op-root $r_3 = r_2 + r_{31}$

$= 2 + 1$

$= \underline{\mathbf{3}}$

The op-root $r_4 = r_2 + r_{41}$

$= 2 + 3$

$= \underline{\mathbf{5}}$

The op-roots are **1**, **2** , **3** and **5**.

The roots are **--1, --2, --3, and –5.**

Example 3.

$X^2+98X+97=0$

(Using theorem 1.)

$$\mathrm{rs} \quad > \quad \frac{\mathrm{K}}{\mathrm{Z}}$$

Z = 98 K = 97

$$\mathrm{rs} \quad > \quad \frac{97}{98}$$

$$> \quad \mathbf{0.9897}$$

$$rs \quad \leq \quad n\frac{\mathrm{K}}{\mathrm{Z}}$$

Z = 98 K = 97 n = 2

$$rs \quad \leq \quad 2 \times \frac{97}{98}$$

$$\leq \quad \mathbf{1.9795}$$

The smallest op-root (rs) is between **0.9897** and **1.9795**(including this number).

The smallest whole number is 1.

Diminish the op-roots by 1.

1	1	98	97
	0	1	97
	1	97	0
	0	1	

1 | 96

The op-root r_1 is **1**.

Part of op-root r_2 = r_{21} = 96

r_2 = r_1 + r_{21}

= 1 + 96

= **97**

The op-roots are **1** and **97**

The roots are **–1 and --97**

--

Example 4.

$X^3—2X^2—X+2=0$ ---------------- equation 1.

This equation has **both negative and positive** op-roots.

We have to change this equation to **all positive op-roots** equation by using Theorem 3.

(Using theorem 3.)

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

A = --2 B = --1 n = 3 $T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

$$rs \geq \frac{-2-(3-1)\sqrt{(-2^2)-3\times(-1)}}{3}$$

$\geq$ **--2.4305**

If we increase the op-roots of the equation by 2.4305.(round it off to 3), the new equation will be all positive op-roots equation.

So increase the op-roots by 3.

3	1	-2	-1	2
	0	3	3	6
	1	1	2	8
	0	3	12	
	1	4	14	
	0	3		
	1	7		

The new equation is

$X^3+7X^2+14X+8=0$ -------------equation 2.

<u>(Using theorem 1.)</u>

$$rs > \frac{K}{Z}$$

Z = 14   K = 8

$$rs > \frac{8}{14}$$

$$> \mathbf{0.5714}$$

$$rs \leq n\frac{K}{Z}$$

Z = 14 K = 8 n = 3

$$rs \leq 3 \times \frac{8}{14}$$

$$\leq \mathbf{1.7142}$$

The smallest op-root (rs) is between **0.5714** and **1.7142**(including this number).

The smallest whole number is 1.

Diminish the op-roots by 1.

1	1	7	14	8
	0	1	6	8
	1	6	8	0
	0	1	5	
	1	5	3	
	0	1		
	1	4		

The op-root r_1 is **1**

The new equation is

$X^2+4X+3=0$ ---------------equation 3

Part of op-root $r_2 = r_{21}$ Part of op-root $r_3 = r_{31}$

Using quadratic equation formula,

$r_{21} = 1$ $r_{31} = 3$

$r_2 = r_1 + r_{21}$

$= 1 + 1$

$= \underline{\mathbf{2}}$

$r_3 = r_1 + r_{31}$

$= 1 + 3$

$= \underline{\mathbf{4}}$

The op-roots of the equation 2 .

$\mathbf{r_1 = \underline{1} \quad r_2 = \underline{2}}$ **and** $\mathbf{r_3 = \underline{4}}$

The op-roots of the original equation (equation 1.)

r_1 -- 3 = 1—3 = **<u>--2</u>**

r_2 -- 3 = 2—3 = **<u>--1</u>**

r_3 -- 3 = 4—3 = **<u>1</u>**

The roots of the original equation (equation 1.)

= **<u>2</u> , <u>1</u> and <u>--1</u>**

Example 5.

$X^5+10X^4+40X^3+80X^2+80X+32=0$

(when $\frac{nK}{Z} = \frac{A}{n}$ **or** $\frac{K}{Z-2Y\frac{K}{Z}} = \frac{A}{n}$ **or** $\frac{nK}{Z} = \frac{K}{Z-2Y\frac{K}{Z}}$ **or** $A^2 - TB = 0$ **,**

all the op-roots of the equation are same and equal to $\frac{A}{n}$ **).**

A = 10 B = 40 Y = 80 Z = 80 K = 32 n = 5

$T = \frac{2n}{n-1} = \frac{2\times5}{5-1} = \frac{5}{2}$

$\frac{A}{n} = \frac{10}{5} = \mathbf{2}$

$\frac{nK}{Z} = \frac{5\times32}{80} = \mathbf{2}$

$\frac{K}{Z-2Y\frac{K}{Z}} = \frac{32}{80-2\times80\times\frac{32}{80}} = \mathbf{2}$

$A^2 - TB = 10^2 -- \frac{5}{2}\times 40 = \mathbf{0}$

So the op-roots all are same and equal to **2**

The roots are

= --2, --2, --2, --2 and --2

----

4. SOLVING POLYNOMIAL EQUATION (2.)

(The Best Method)

Using Theorems 1,3 and 5, we can solve polynomial equation whose **all roots are real**. We can use other theorems too. Theroem.5 helps to solve equations very, very quickly, shown in example.3

The method **('From top to bottom value approach method ')** which I used here to solve the equation, is the best method. Because it gives quick answer for any kind of equations whose **all roots are real**. The answer is in numerical value.

I have shown here how to find the roots of the following three equations.

1. $X^3 + 14X^2 + 7X -- 78 = 0$
2. $X^5 + 37X^4 + 521X^3 + 3443X^2 + 10518X + 11880 = 0$
3. $X^2 + 190X + 7000 = 0$

Example 1.

$X^3 + 14X^2 + 7X -- 78 = 0$ --------- equation 1.

This equation has **both positive and negative op-roots.** We have to change the equation to **all positive op-roots** equation, using Theorem 3.

(**Using theorem 3.)**

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

$$A = 14 \quad B = 7 \quad n = 3 \quad T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$$

$$rs \geq \frac{14-(3-1)\sqrt{14^2-3\times7}}{3}$$

$$\geq \mathbf{--\ 4.1525}$$

So if we increase the op-roots of the equation by 4.1525,(round it off to 5), the new equation will be **all positive op-roots** equation.

So increase the op-roots of the equation by 5.

5	1	14	7	-78
	0	5	95	510
	1	19	102	432
	0	5	120	
	1	24	222	
	0	5		
	1	29		

The new equation is

$X^3 + 29X^2 + 222X + 432 = 0$ ----------- equation 2.

(Using theorem 1.)

$$rs > \frac{K}{Z}$$

Z = 222 K = 432

$$rs > \frac{432}{222}$$

$$> \mathbf{1.9459}$$

$$rs \leq n\frac{K}{Z}$$

Z = 222 K = 432 n = 3

$$rs \leq 3 \times \frac{432}{222}$$

$$\leq \mathbf{5.8378}$$

The smallest op-root 'rs' is between **1.9459** and **5.8378** (including this number).

In graph(Op-roots Scale)

1.9459	2.9188	3.8918	4.8648	5.8378

|-------------------|-----------------------|----------------------|---------------------|

0	¼	½	¾	1

(From top to bottom value approach method.)

Diminish the op-roots by ¾ of the value = 4.8648(round it of to the nearest whole number = 5).

5	1	29	222	432
×	0	5	120	510
	1	24	102	--78

When minus sign appears, don't proceed further. Mark × Sign to identify it as cancelled.

Diminish the op-roots by ½ of the value = 3.8918 (Round it off to 4).

4	1	29	222	432
×	0	4	100	488
	1	25	122	--56

Don't proceed further. Since minus sign appeared.

Diminish the op-roots by ¼ of the value = 2.9188(Round it off to 3).

3	1	29	222	432
	0	3	78	432
	1	26	144	0
	0	3	69	
	1	23	75	
	0	3		
	1	20		

The op-root r_1 is **3**.

The new equation is $X^2 + 20X + 75 = 0$ ------------- equation 3.

Part of op-root $r_2 = r_{21}$ part of op-root $r_3 = r_{31}$

Using Quadratic equation formula,

$r_{21} = 5$ $r_{31} = 15$

op-root $r_2 = r_1 + r_{21} = 3 + 5 =$ **8**

op-root $r_3 = r_1 + r_{31} = 3 + 15 =$ **18**

The op-roots of the equation (2) are **3**,**8** and **18**.

The op-roots of the original equation (equation 1.) is

r_1 -- 5 = 3 – 5 = **--2**

r_2 -- 5 = 8 – 5 = **3**

r_3 -- 5 = 18 – 5 = **13**

The roots of the original equation (equation1) are **2,--3 and –13** .

Example 2.

$X^5 + 37X^4 + 521X^3 + 3443X^2 + 10518X + 11880 = 0$ -----------equation 1.

The equation's op-roots **all are positive.** So we need not use 'Theorem 3'.

(Using theorem 1.)

$$rs > \frac{K}{Z}$$

Z = 10518 K = 11880

$$rs > \frac{11880}{10518}$$

$$> \mathbf{1.1294}$$

$$rs \leq n\frac{K}{Z}$$

Z = 10518 K = 11880 n = 5

$$rs \leq 5 \times \frac{11880}{10518}$$

$$\leq \mathbf{5.6474}$$

The smallest op-root (rs) is between **1.1294** and **5.6474**(including this number).

In graph(Op-roots scale)

1.1294	2.2589	3.3884	4.5179	5.6474

|-----------------|--------------------|---------------------|----------------------|

0	1/4	1/2	3/4	1

(‘From top to bottom approach method’)

Diminish the op-roots by ¾ of the value = 4.5179(round it of to 5).

5	1	37	521	3443	10518	11880
×	0	5	160	1805	8190	11640
	1	32	361	1638	2328	240
	0	5	135	1130	2540	
	1	27	226	508	**--212**	

Don’t proceed further. Since minus sign appeared. Mark × sign to identify it as cancelled.

Diminish the op-roots by ½ of the value = 3.3884(round it off to 3).

3	1	37	521	3443	10518	11880
	0	3	102	1257	6558	11880
	1	34	419	2186	3960	0
	0	3	93	978	3624	
	1	31	326	1208	336	
	0	3	84	726		
	1	28	242	482		
	0	3	75			
	1	25	167			

0	3
1	22

The op-root r_1 is **<u>3</u>**.

The new equation is

$X^4 + 22X^3 + 167X^2 + 482X + 336 = 0$ ---------------- equation 2.

$$rs > \frac{K}{Z}$$

Z = 482   K = 336

$$rs > \frac{336}{482}$$

$$> \mathbf{0.6970}$$

$$rs \leq n\frac{K}{Z}$$

Z = 482   K = 336   n = 4

$$rs \leq 4 \times \frac{336}{482}$$

$$\leq \mathbf{2.7883}$$

The smallest op-root (rs) is between **0.6970** and **2.7883** (including this number)

<u>In graph(Op-roots Scale)</u>

0.6970   1.2198   1.7426   2.2654   2.7883

|-------------------|------------------------|-----------------------|---------------------|

0   ¼   ½   ¾   1

Diminish the op-roots by ¾ of the value = 2.2654(Round it off to 2).

2	1	22	167	482	336
×	0	2	40	254	456
	1	20	127	228	**--120**

Don't proceed further. Since minus sign appeared.

Diminish the op-roots by ½ of the value = 1.7426(Round it off to 1).

1	1	22	167	482	336
	0	1	21	146	336
	1	21	146	336	0
	0	1	20	126	
	1	20	126	210	
	0	1	19		
	1	19	107		
	0	1			
	1	18			

Part of the op-root $r_2 = r_{21} =$ **1**

The op-root $r_2 = r_1 + r_{21}$

= 3+1 = **4**

The new equation is

$X^3 + 18X^2 + 107X + 210 = 0$ --------------------equation 3.

$$rs > \frac{K}{Z}$$

Z = 107 K = 210

$$rs > \frac{210}{107}$$

$$> \mathbf{1.9626}$$

$$rs \leq n\frac{K}{Z}$$

Z =107   K = 210   n = 3

$$rs \leq 3 \times \frac{210}{107}$$

$$\leq \mathbf{5.8878}$$

<u>In graph(Op-roots Scale)</u>

1.9626	2.9439	3.9252	4.9065	5.8878
0	¼	½	¾	1

Diminish the op-roots by ¾ of the value = 4.9065(Round it off to 5).

5	1	18	107	210
	0	5	65	210
	1	13	42	0
	0	5	40	
	1	8	2	
	0	5		
	1	3		

Part of op-root $r_3 = r_{31} = 5$

The op-root $r_3 = r_2 + r_{31}$

$= 4 + 5 =$ **<u>9</u>**

The new equation is

$$X^2 + 3X + 2 = 0$$

Part of op-root $r_4 = r_{41}$

Part of op-root $r_5 = r_{51}$

Using Quadratic Equation Formula,

$r_{41} =$ **1** $r_{51} =$ **2**

The op-root $r_4 = r_3 + r_{41}$

$= 9 + 1$

$=$ **<u>10</u>**

The op-root $r_5 = r_3 + r_{51}$

$= 9 + 2$

$=$ **<u>11</u>**

The op-roots of the equation (1.) r_1, r_2, r_3, r_4 & r_5 = **<u>3</u>, <u>4</u>, <u>9</u>, <u>10</u> & <u>11</u>**

The roots are= **<u>--3</u>,-<u>-4</u>,--<u>9</u>,--<u>10</u> & --<u>11</u>**

<u>Example 3.</u>

$$X^2 + 190X + 7000 = 0$$ ------------------equation 1.

<u>(Using Theorem 1.)</u>

$$rs \quad > \quad \frac{K}{Z}$$

Z = 190 K = 7000

$$rs \quad > \quad \frac{7000}{190}$$

$$> \quad \mathbf{36.8421}$$

$$rs \quad \leq \quad n\frac{K}{Z}$$

Z = 190 K = 7000 n= 2

$$rs \quad \leq \quad 2 \times \frac{7000}{190}$$

$$\leq \quad \mathbf{73.6842}$$

<u>In graph(Op-roots Scale)</u>

36.8421	46.0526	55.2631	64.4736	73.6842
|------------------	|----------------------	|---------------------	|--------------------|	
0	¼	½	¾	1

<u>(Using 'From top to bottom value approach method'.)</u>

Diminish the op-roots by ¾ of the value equal to 64.4736(round it of to the nearest whole number equal to 64.)

64	1	190	7000
×	0	64	8064
	1	126	**--1064**

Don't proceed further .Since minus sign appeared. Mark × sign to identify it as cancelled.

Diminish the op-roots by ½ of the value = 55.2631(round it off to 55.)

55	1	190	7000
×	0	55	7425
	1	135	--425

Don't proceed further. Since minus sign appeared.

Diminish the op-root by ¼ of the value = 46.0526(round it off to 46.)

46	1	190	7000
	1	46	6624
	1	144	376
	0	46	
	1	98	

Part of op-root $r_1 = r_{11} =$ **46**

The new equation is

$X^2 + 98X + 376 = 0$ ---------------------equation 2.

$$rs > \frac{K}{Z}$$

Z = 98 K = 376

$$rs > \frac{376}{98}$$

$$> \mathbf{3.8367}$$

$$rs \leq n\frac{K}{Z}$$

Z = 98 K = 376 n = 2

$$rs \leq 2 \times \frac{376}{98}$$

$$\leq \mathbf{7.6734}$$

<u>In graph(Op-roots Scale)</u>

3.8367 4.7958 × 5.7550 × 6.7142 7.6734

|-------------------|------------------------|-----------------------|---------------------|

0 ¼ ½ ¾ 1

<u>(Using Theorem 5.)</u>

As per Theorem 5, the smallest op-root (rs) value must be below ½ of the value.**(refer theorem 5.)**

Diminish the op-roots by ¼ of the value = 4.7958(round it of to the nearest whole number 5.)

5	1	98	376
×	0	5	465
	1	93	**--89**

Don't proceed further .Since minus sign appeared.

Diminish the op-root by more than the starting value 3.8367(Round it off to 4).

4	1	98	376
	0	4	376
	1	94	0
	0	4	
	1	90	

Another part of $r_1 = r_{12} =$ **4**

The op-root $r_1 = r_{11} + r_{12} = 46 + 4 =$ **<u>50</u>**

Part of op-root $r_2 = r_{21} =$ **90**

The op-root $r_2 = r_1 + r_{21} = 50 + 90 =$ **<u>140</u>**

The op-roots r_1, and r_2 = **50, & 140** .

The roots are= **<u>--50</u>, <u>--140</u>**

----

5. THEOREM 6

Theorem 6.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X. (n = Positive whole number & > 1.)

Any op-root 'r'of the above equation is equal to $\frac{K}{Z} + (n-1)\frac{K}{Z}R$ where' **R'** is Effective Ratio of that op-root ' r '. **(Effective Ratio -- see chapter 8 ' Definitions .')**

op-root ' r ' = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$

<u>Example</u>

$X^3 + 15X^2 + 71X + 105 = 0$

The op-roots are 3, 5 and 7 .

Effective Ratio of op-root ' a ' = $(\frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e}$ - - - - - - - - - - $)\frac{a}{n-1}$

a = 5 b = 3 c = 7 n = 3

Effective Ratio(R) of the op-root ' 5 ' = $\left(\frac{1}{3}+\frac{1}{7}\right)\frac{5}{3-1}$

$$R = \mathbf{\frac{25}{21}}$$

$$\text{op-root ' r '} = \frac{K}{Z} + (n-1)\frac{K}{Z}R$$

Z = 71 K = 105 n = 3 R = $\mathbf{\frac{25}{21}}$ op-root'r' = op-root'a'

$$\text{op-root ' a '} = \frac{105}{71} + (3-1)\frac{105}{71} \times \frac{25}{21}$$

op-root ' 5 ' = **<u>5</u>**

<u>6 . THEOREMS 7 TO 10</u>

(The Biggest Op-root.)

<u>Theorem 7.</u>

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X (n = Positive whole number & > 1.)

The biggest op-root (rb) of the above equation is more than or equal to $\frac{A}{n}$, and less than A. *(The biggest op-root= more precisely the op-root which is not smaller than any other op-root in the equation.)*

$$rb \geq \frac{A}{n}$$

& $$rb < A$$

Example .

$X^3+12X^2+39X+28 = 0$

$$rb \geq \frac{A}{n}$$

A = 12 n = 3

$$rb \geq \frac{12}{3}$$

$$\geq \mathbf{4}$$

$$rb < A$$

A = 12

$$rb < \mathbf{12}$$

The biggest op-root (rb) is between **4** (including this number) and **12**

Theorem 8.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2

and X respectively. K is a constant. And n is the highest power of X (n = Positive whole number & > 1.)

The biggest op-root (rb) of the above equation is more than or equal to $A - \frac{2B}{A}$ and less than $A - \frac{B}{A}$.

$$rb \geq A - \frac{2B}{A}$$

& $$rb < A - \frac{B}{A}$$

Example.

$X^3 + 12X^2 + 39X + 28 = 0$

$$rb \geq A - \frac{2B}{A}$$

A = 12 B = 39

$$rb \geq 12 - \frac{2 \times 39}{12}$$

$$\geq \underline{5.5}$$

$$rb < A - \frac{B}{A}$$

A = 12 B = 39

$$rb < 12 - \frac{39}{12}$$

$$< \underline{\mathbf{8.75}}$$

The biggest op-root (rb) is between **5.5** (including this number) and **8.75**

Theorem 9.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real (negative or positive or both)**. A, B, Y and Z are co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X.(n = Positive whole number & > 1.)

The biggest op-root (rb) of the above equation is more than or equal to $\frac{A+\sqrt{A^2-TB}}{n}$ and less than or equal to $\frac{A+(n-1)\sqrt{A^2-TB}}{n}$.

{ THE VALUE 'T' = $\frac{2n}{n-1}$ }

$$rb \geq \frac{A+\sqrt{A^2-TB}}{n}$$

$$\& \quad rb \leq \frac{A+(n-1)\sqrt{A^2-TB}}{n}$$

Example X^3 -- X^2 -- 8X+12 = 0

$$rs \geq \frac{A+\sqrt{A^2-TB}}{n}$$

A = -- 1 B = -- 8 n = 3 $T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

rs $\geq \dfrac{-1+\sqrt{(-1^2)-(3\times-8)}}{3}$

$\geq$ **<u>1.3333</u>**

rs $\leq \dfrac{A+(n-1)\sqrt{A^2-TB}}{n}$

A = -- 1 B = -- 8 n = 3 T = 3

rs $\leq \dfrac{-1+(3-1)\sqrt{(-1^2)-(3\times-8)}}{3}$

$\leq$ **<u>3</u>**

The biggest op-root (rb) is between **<u>1.3333</u>** and **<u>3</u>** (including both numbers)

<u>Theorem 10.</u>

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X. (n = Positive whole number & > 1.)

The biggest op-root (rb) of the above equation is more than or equal to $\dfrac{nK}{Z-\sqrt{Z^2-TYK}}$ and less than or equal to $\dfrac{nK}{Z-(n-1)\sqrt{Z^2-TYK}}$.

{ THE VALUE 'T' = $\dfrac{2n}{n-1}$ }

$$rb \geq \frac{nK}{Z-\sqrt{Z^2-TYK}}$$

& $$rb \leq \frac{nK}{Z-(n-1)\sqrt{Z^2-TYK}}$$

Example.

$X^3 + 15X^2 + 56X + 60 = 0$

$$rb \geq \frac{nK}{Z-\sqrt{Z^2-TYK}}$$

Y = 15 Z = 56 K = 60 n = 3 $T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

$$rb \geq \frac{3\times60}{56-\sqrt{56^2-3\times15\times60}}$$

$$\geq \underline{5.1253}$$

$$rb \leq \frac{nK}{Z-(n-1)\sqrt{Z^2-TYK}}$$

Y = 15 Z = 56 K = 60 n = 3 T = 3

$$rb \leq \frac{3\times60}{56-(3-1)\sqrt{56^2-3\times15\times60}}$$

$$\leq \underline{12.6415}$$

The biggest op-root (rb) is between **5.1253** and **12.6415**(including both numbers)

7.SOLVING POLYNOMIAL EQUATION (3.)

(The Biggest Op-root.)

Finding the biggest op-root is a little difficult than finding the smallest op-root. So it is not interesting. However here is an example.

Example.

$X^3+10X^2+29X+20=0$ --------------- equation 1.

(Using Theorem 8.)

$$rb \geq A - \frac{2B}{A}$$

A = 10 B = 29

$$rb \geq 10 - \frac{2\times29}{10}$$

$$\geq \mathbf{4.2}$$

$$rb < A - \frac{B}{A}$$

A = 10 B = 29

$$rb < 10 - \frac{29}{10}$$

$$< \mathbf{7.1}$$

The biggest op-root (rb) is between **4.2** (including this number) and **7.1**

Diminish the op-root by the average value $= \frac{4.2+7.1}{2} =$5.65 (round it off to 6).

6	1	10	29	20
×	0	6	24	30
	1	4	5	--10
	0	6	--12	
	1	--2	17	
	0	6		
	1	--8		

6 is more than the biggest op-root(rb).

Diminish the op-roots by the average value = $\frac{4.2+6}{2} = 5.1$(round it off to 5).

5	1	10	29	20
	0	5	25	20
	1	5	4	0

The op-root r_1 = **5**

The new equation is

$X^2+5X+4 = 0$

Using Quadratic Equation Formula.

r_2 = **4** r_3 = **1**

The op-roots are r_1 = **5** r_2 = **4** and r_3 = **1**

The roots are,

r_1 = --5 r_2 = --4 and r_3 = --1

8.DEFINITIONS.

1.Effective Ratio(R).

Let a, b, c, d, e, - - - - - - - - - -are the op-roots of an equation, whose **all op-roots are real and positive**.

Effective Ratio (**R**) of the op-root ‘ **a** ‘ = $(\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}- - - -)\frac{a}{n-1}$

Example.

$X^3+15X^2+56X+60=0$

The op-roots are 2, 3 and 10.

Effective Ratio (**R**) of the op-root ‘**a** ‘= $(\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}- - - -)\frac{a}{n-1}$

a = 2 b = 3 c = 10

Effective Ratio (**R**) of the op-root ‘ **2** ‘ = $(\frac{1}{3}+\frac{1}{10})\frac{2}{3-1}$

$= \mathbf{\frac{13}{30}}$

2.The value of T.

$$T = \frac{2n}{n-1} \qquad \text{(n is positive whole number.)}$$

<u>Example.</u>

$$T = \frac{2n}{n-1}$$ Let **n** (the highest power of an equation) is 5.

$$T = \frac{2 \times 5}{5-1}$$

= **<u>2.5</u>**

<u>3. Identifying co-efficients</u>.

A B

1. $(1)X^2 + L\,X + K = 0$

Y Z K

A = L B = K Y = 1 Z = L K = K

A B

2. $X^3 + L\,X^2 + M\,X + K = 0$

Y Z K

A = L B = M Y = L Z = M K = K

A B

3. $X^4 + L\,X^3 + M\,X^2 + N\,X + K = 0$

Y Z K

A = L B = M Y = M Z = N K = K

4. $X^5 + L X^4 + M X^3 + N X^2 + P X + K = 0$

(L → A, M → B, N → Y, P → Z, K → K)

A = L B = M Y = N Z = P K = K

5. $X^6 + LX^5 + M X^4 + N X^3 + P X^2 + Q X + K = 0$

(L → A, M → B, P → Y, Q → Z, K → K)

A = L B = M Y = P Z = Q K = K

---

4. **Multiplication Number (MN) .**(in the equation whose **all op-roots are real and positive**.)

Multiplication Number (**MN**) of op-root ‘ **a** ‘ = $\frac{Z-\frac{K}{a}}{Y}$

Example.

$X^3+15X^2+66X+80=0$

The op-roots are 5, 2 and 8.

Multiplication Number (MN) of op-root ‘ a ‘ = $\frac{Z-\frac{K}{a}}{Y}$

a = 5 Y = 15 Z = 66 K = 80

Multiplication Number (MN) of op-root ‘ 5 ‘ = $\frac{66-\frac{80}{5}}{15}$

$= \mathbf{\frac{10}{3}}$

---

5. **<u>‘ MN ‘ Op-root Formula</u>**.(in the equation whose **all op-roots are real and positive**.)

$$\text{op-root ‘ r ‘} = \frac{K}{Z - Y \times (\text{MN of op-root ‘r’})}$$

<u>Example.</u>

$X^3+15X^2+66X+80=0$

The op-roots are 2, 5 and 8

$$\text{‘MN’ of op-root ‘ a ‘} = \frac{Z - \frac{K}{a}}{Y}$$

Y = 15   Z = 66   K = 80   a = 2

$$\text{‘MN’ of op-root ‘ 2 ‘} = \frac{66 - \frac{80}{2}}{15}$$

$$\text{‘MN’ of op-root ‘2 ‘} = \frac{26}{15}$$

$$\text{op-root ‘ r ‘} = \frac{K}{Z - Y \times (\text{MN of op-root r})}$$

Y = 15   Z = 66   K = 80   MN = $\frac{26}{15}$

$$\text{op-root ‘ a ‘} = \frac{80}{66 - 15 \times \frac{26}{15}}$$

op-root ‘ a ‘ = **<u>2</u>**

----

6. Extra .

The value of A^2 –TB remains unchanged

(In the equation whose **all op-roots are real or imaginary and negative or positive or both**.)

The value of **A^2 –TB** of a new equation, which may be obtained by **diminishing or increasing** the original equation, by **any number**, is **same** as the value of **A^2 –TB** of the original equation.

Example1.

$X^3 --13X^2 +31X + 45 = 0$ ----------------- equation.1

A = --13 B = 31 $T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3$

$A^2 -TB = (--13)^2 — (3\times31) = \underline{\mathbf{76}}$

Increase the op-roots by 6.

6	1	--13	31	45
	0	6	--42	--66
	1	--7	--11	--21
	0	6	--6	
	1	--1	--17	
	0	6		
	1	5		

The new equation = $X^3 +5X^2 --17X --21 = 0$ ------------- equation.2

A = 5 B = --17

$A^2 -TB = 5^2 — (3\times(--17)) = \underline{\mathbf{76}}$

(A^2 –TB of equation.1) **76** = **76** (A^2 –TB of equation.2)

--

<u>Example 2.</u>

$X^4+7X^3+18X^2+20X+8 = 0$ -------------------- equation.1

A = 7 B = 18 $T = \frac{2n}{n-1} = \frac{2\times4}{4-1} = \frac{8}{3}$

$A^2 - TB = 7^2 - \frac{8}{3}\times18$

= **<u>1</u>**

Diminish the op-roots by 3.

3	1	7	18	20	8
	0	3	12	18	6
	1	4	6	2	2
	0	3	3	9	
	1	1	3	--7	
	0	3	--6		
	1	--2	9		
	0	3			
	1	--5			

The new equation is

$X^4 - 5X^3 + 9X^2 - 7X + 2 = 0$ --------------- equation.2

A = --5 B = 9 $T = \frac{8}{3}$

$A^2 - TB = (--5)^2 - \frac{8}{3} \times 9$

= **<u>1</u>**

(A^2 –TB of equation.1) **1** = **1** (A^2 –TB of equation.2)

<u>9. PROOF.</u>

Although these theorems are proved, when **'n'**,(the highest power of the equation) is equal to **3**, It is also applicable to all values of n (n = Positive whole number & > 1.).There may be another better way of proving these theorems.

<u>1. Theorem 1. & Proof.</u>

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than $\frac{K}{Z}$ and less than or equal to n$\frac{K}{Z}$. *(The smallest* op-root *(rs) = more precisely, the op-root which is not bigger than any other op-root in the equation.)*

$$rs > \frac{K}{Z}$$

$$\& \quad rs \leq n\frac{K}{Z}$$

<u>Proof:</u>

Consider the following equation $X^3 + YX^2 + ZX + k = 0$,

whose **all op-roots are real and positive**.

a, b, & c are the op-roots of the equation.

$X^3 + (a+b+c) X^2 + (ab + bc + ca) X + abc = 0$

Let **'a'** is the smallest op-root.

(a).

$$rs > \frac{K}{Z}$$

rs = a   Z = ab + bc + ca   K = abc

$$a > \frac{abc}{ab+bc+ca}$$

Divide both side by a

$$1 > \frac{bc}{ab+bc+ca}$$

ab + bc + ca > bc

$$rs > \frac{K}{Z}$$ (**proved.)**

(b)

$$rs \leq n\frac{K}{Z}$$

rs = a   Z = ab + bc + ca   K = abc   n =3

$$a \leq 3 \times \frac{\mathrm{abc}}{\mathrm{ab+bc+ca}}$$

Divide both side by a

$$1 \leq 3 \times \frac{\mathrm{bc}}{\mathrm{ab+bc+ca}}$$

ab + bc + ca $\leq$ 3bc

$$\boldsymbol{rs} \leq \boldsymbol{n}\frac{\mathbf{K}}{\mathbf{Z}}$$ **(Proved.)**

Theorem 2. & Proof.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than $\frac{K}{Z-Y\frac{K}{Z}}$ and less than or equal to $\frac{K}{Z-2Y\frac{K}{Z}}$.

$$rs > \frac{K}{Z-Y\frac{K}{Z}}$$

$$\& \qquad rs \leq \frac{K}{Z-2Y\frac{K}{Z}}$$

Proof.

Consider the following equation $X^3 + YX^2 + ZX + K = 0$,

Whose **all op-roots are real and positive.**

a, b, & c are the op-roots of the equation.

$X^3 + (a+b+c)X^2 + (ab + bc + ca) X + abc = 0$

Let ‘a’ is the smallest op-root.

(a)

$$rs > \frac{K}{Z-Y\frac{K}{Z}}$$

It is enough to prove $\frac{K}{Z}$ is less than $\frac{ab+ac}{a+b+c}$

(Since op-root ‘a’ = $\frac{K}{Z-Y(\text{MN of op}-\text{root a})}$)

(MN of op-root ‘a’ = $\frac{ab+ac}{a+b+c}$)

$$\frac{K}{Z} < \frac{ab+ac}{a+b+c}$$

$$\frac{abc}{ab+bc+ca} < \frac{ab+ac}{a+b+c}$$

Divide both side by ‘a’

$$\frac{bc}{ab+bc+ca} < \frac{b+c}{a+b+c}$$

$abc + b^2c + c^2b < b^2a + b^2c + c^2a + c^2b + 2abc$

$0 < b^2a + c^2a + abc$ $\quad rs > \frac{K}{Z-Y\frac{K}{Z}}$ **(Proved.)**

(b)

$$rs \leq \frac{K}{Z-2Y\frac{K}{Z}}$$

It is enough to prove $\frac{2K}{Z}$ is more than or equal to $\frac{ab+ac}{a+b+c}$

$$\frac{2K}{Z} \geq \frac{ab+ac}{a+b+c}$$

$$\frac{2abc}{ab+bc+ca} \geq \frac{ab+ac}{a+b+c}$$

Divide both side by 'a'

$$\frac{2bc}{ab+bc+ca} \geq \frac{b+c}{a+b+c}$$

$$2abc + 2b^2c + 2c^2b \geq b^2a + b^2c + c^2a + c^2b + 2abc$$

$$b^2c + c^2b \geq b^2a + c^2a$$ **(proved.).**

So,

$$rs \leq \frac{K}{Z-2Y\frac{K}{Z}}$$

Theorem – 3 & Proof.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real (negative or positive or both)**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And 'n' is the highest power of X (n = Positive whole number & > 1.)

The smallest op-root (rs) of the above equation is more than or equal to $\frac{A-(n-1)\sqrt{A^2-TB}}{n}$ and less than or equal to $\frac{A-\sqrt{A^2-TB}}{n}$.

{ THE VALUE 'T' = $\frac{2n}{n-1}$ }

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

$$\& \qquad rs \leq \frac{A-\sqrt{A^2-TB}}{n}$$

Proof.

Consider the following equation $X^3 + AX^2 + BX + K= 0$,

whose **all op-roots are real (negative or positive or both).**

Let the op-roots are r, r+s & r + s+ t.

r is the smallest op-root.

s and t are positive number(or 0).

$X^3 + (3r + 2s + t) X^2 + (3r^2 + 4rs + 2rt + s^2 + st)X + K = 0$

(a)

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

$A = 3r + 2s + t$ $\qquad$ $B = 3r^2 + 4rs + 2rt + s^2 + st$

$$T = \frac{2n}{n-1} = \frac{2\times3}{3-1} = 3 \qquad n = 3$$

$$rs \geq \frac{(3r + 2s + t)-(3-1)\sqrt{(3r + 2s + t)^2-3(3r^2+ 4rs + 2rt + s^2 + st)}}{3}$$

$$rs \geq \frac{(3r + 2s + t)-\sqrt{4s^2+4st+4t^2}}{3}$$

If we subtract 2s+t or $\sqrt{4s^2 + 4st + t^2}$ from (3r + 2s + t), we get the answer 'r' (the smallest op-root(rs)). But actually we subtract $\sqrt{4s^2 + 4st + 4t^2}$ which is more than or equal to $\sqrt{4s^2 + 4st + t^2}$. So the answer must be less than or equal to 'r' (the smallest op-root (rs)).

So,

$$rs \geq \frac{A-(n-1)\sqrt{A^2-TB}}{n}$$

(b)

$$rs \leq \frac{A-\sqrt{A^2-TB}}{n}$$

$$rs \leq \frac{(3r + 2s + t) - \sqrt{(3r + 2s + t)^2 - 3(3r^2 + 4rs + 2rt + s^2 + st)}}{3}$$

$$rs \leq \frac{(3r + 2s + t)-\sqrt{s^2+st+t^2}}{3}$$

If we subtract 2s+t or $\sqrt{4s^2 + 4st + t^2}$ from (3r + 2s + t), we get the answer 'r' (the smallest op-root(rs)). But actually we subtract $\sqrt{s^2 + st + t^2}$ which is less than or equal to$\sqrt{4s^2 + 4st + t^2}$. So the answer must be more than or equal to 'r' (the smallest op-root (rs)).

$$rs \leq \frac{A-\sqrt{A^2-TB}}{n}$$

Theorem 9. & Proof.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real (negative or positive or both)**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And 'n' is the highest power of X (n = Positive whole number & > 1.)

The biggest op-root (rb) of the above equation is more than or equal to $\frac{A+\sqrt{A^2-TB}}{n}$ and less than or equal to $\frac{A+(n-1)\sqrt{A^2-TB}}{n}$. *(The biggest op-root= more precisely the op-root which is not smaller than any other op-root in the equation.)* **{ THE VALUE 'T' = $\frac{2n}{n-1}$ }**

$$rb \geq \frac{A+\sqrt{A^2-TB}}{n}$$

$$\& \quad rb \leq \frac{A+(n-1)\sqrt{A^2-TB}}{n}$$

Proof.

Consider the following equation $X^3 + AX^2 + BX + k = 0$,

whose **all op-roots are real (negative or positive or both).**

Let the op-roots are r, r+s & r + s+ t.

r + s + t is the biggest op-root.

s and t are positive number (or 0).

$$X^3+ (3r + 2s + t) X^2+(3r^2 + 4rs + 2rt + s^2 + st)X+ K = 0$$

(a)

$$rb \quad \geq \quad \frac{A+\sqrt{A^2-TB}}{n}$$

A = 3r + 2s + t B = $3r^2 + 4rs + 2rt + s^2 + st$

$$T = \frac{2n}{n-1} = \frac{2\times 3}{3-1} = 3 \qquad n = 3$$

$$rb \quad \geq \quad \frac{(3r + 2s + t)+\sqrt{(3r + 2s + t)^2-3(3\,r^2+ 4rs + 2rt + s^2+ st)}}{3}$$

$$\geq \quad \frac{(3r + 2s + t)+\sqrt{s^2+st+t^2}}{3}$$

If we add s+2t or $\sqrt{s^2 + 4st + 4t^2}$ to (3r + 2s + t) , we get answer 'r+ s + t' (the biggest op-root(rb)). But actually we add $\sqrt{s^2 + st + t^2}$ which is less than or equal to $\sqrt{s^2 + 4st + 4t^2}$. So the answer must be less than or equal to 'r + s + t' (the biggest op-root (rb)).

So,

$$rb \quad \geq \quad \frac{A+\sqrt{A^2-TB}}{n}$$

(b)

$$rb \quad \leq \quad \frac{A+(n-1)\sqrt{A^2-TB}}{n}$$

$$rb \quad \leq \quad \frac{(3r + 2s + t)+(n-1)\sqrt{(3r + 2s + t)^2-3(3r^2 + 4rs + 2rt + s^2+ st)}}{3}$$

$$\leq \quad \frac{(3r + 2s + t)+\sqrt{4s^2+4st+4t^2}}{3}$$

If we add s+2t or $\sqrt{s^2 + 4st + 4t^2}$ to $(3r + 2s + t)$, we get answer 'r+ s + t' (the biggest op-root(rb)). But actually we add $\sqrt{4s^2 + 4st + 4t^2}$ which is more than or equal to $\sqrt{s^2 + 4st + 4t^2}$. So the answer must be more than or equal to 'r + s + t' (the biggest op-root (rb)).

So,

$$rb \quad \leq \quad \frac{A+(n-1)\sqrt{A^2-TB}}{n}$$

Theorem 5. – Proof.

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive**. A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X .(n = Positive whole number & > 1.)

Effective ratio (NrsR) of the smallest op-root (Nrs) of a new equation, which is obtained by diminishing the op-roots of the above original equation by a positive number,(which is less than the smallest op-root(rs) of the original equation)) is less than Effective Ration (rsR) of the smallest op-root (rs) of the original equation(Effective Ratio – see 'Definitions' chapter.)

$$\text{NrsR} \quad < \quad \text{rsR}$$

Proof.

Consider the following equation $X^3 + YX^2 + ZX + K = 0$, whose **all op-roots are real and positive.** Let the op-roots are a, b, & c.

Effective Ratio (**aR**) of the smallest op-root 'a' $=\left(\frac{a}{b}+\frac{a}{c}\right)\times\frac{1}{2}$

After diminishing the op-roots, by a number's'.

Effective Ratio ('a –s' R) of the smallest op-root (a-s) $=\left(\frac{a-s}{b-s}+\frac{a-s}{c-s}\right)\times\frac{1}{2}$

$$\left(\frac{a-s}{b-s}+\frac{a-s}{c-s}\right)\times\frac{1}{2} \quad < \quad \left(\frac{a}{b}+\frac{a}{c}\right)\times\frac{1}{2}$$

$$\left(\frac{a-s}{b-s}+\frac{a-s}{c-s}\right) \quad < \quad \left(\frac{a}{b}+\frac{a}{c}\right)$$

When a positive number (which is less than numerator), is subtracted from both the numerator and the denominator of a positive fraction (which is less than one), the new fraction is less than the original fraction.

$$\left(\frac{a-s}{b-s}+\frac{a-s}{c-s}\right) \quad < \quad \left(\frac{a}{b}+\frac{a}{c}\right)$$ **(proved.)**

Theorem 6. –Proof

In an equation, $X^n+AX^{n-1}+BX^{n-2}$- - - - - - - - - - - - - - $YX^2+ZX+K = 0$, whose **all op-roots are real and positive.** A, B, Y and Z are the co-efficient of X^{n-1}, X^{n-2}, X^2 and X respectively. K is a constant. And n is the highest power of X.(n = Positive whole number & > 1.)

Any op-root'r'of the above equation is equal to $\frac{K}{Z}+(n-1)\frac{K}{Z}R$ where R is Effective Ratio of that op-root. (Effective Ratio- see chapter8'Definitions '.)

op-root **' r '** = $\frac{K}{Z}+(n-1)\frac{K}{Z}R$

Proof.

Consider the following equation $X^3 + YX^2 + ZX + K = 0$, whose **all op-roots are real and positive.** Let the op-roots are a, b, & c.

$X^3 + (a+b + c)X^2 + (ab + bc + ca)X + abc = 0$

op-root ‘ r ‘ = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$ where R is Effective Ratio of op-root ‘r’.

r = a Z = ab + bc + ca K = abc R = aR

Effective Ratio (aR) of op-root ‘a’ = $\frac{ab+ac}{2bc}$

op-root ‘ r ‘ = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$

op-root’a’ = $\frac{K}{Z}(1 + (n-1)R\ )$

= $\frac{K}{Z}\{\ 1 + (2 \times \frac{ab+ac}{2bc}\ )\ \}$

= $\frac{K}{Z}\ (\frac{ab+ac+bc}{bc}\ ) = \frac{abc}{ab+ac+bc} \times \frac{ab+ac+bc}{bc}$

op-root ’a’ = **’ a’**

op-root ‘ r ‘ = $\frac{K}{Z} + (n-1)\frac{K}{Z}R$ **(PROVED)**

Proof For Other Theorems .

Proof for theorem 4,7,8 & 10 .

The original reference equation is to be changed to another equation, whose op-roots are the reciprocal of the op-roots of the original reference equation. Then, the Theorems 4, 7, 8 & 10 can be proved like Theorems 9, 1, 2 & 3 respectively.

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<u>10 . CONCLUSION.</u>

In example sections, I explained only few methods of solving polynomial equations. There are many other options and methods available.

In part2, there are five theorems. They are the same type as previous theorems but a different. It has many new definitions, which are really new and has **<u>one important finding.</u>**

Part 2 will be written soon, if this (part 1) is appreciated by you. **Perhaps those theorems in Part 2, may pave the way to find the op-roots in a straight method**.

*****END*****

---(05.02.2009.OF)

<u>For any clarification, or your comments, please write to,</u>

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